Chapter 6

- Introduction
- Concepts of stress and strain
- Stress-strain behavior
- Anelasticity
- Elastic properties of materials
- Tensile properties
- True stress and strain
- Elastic recovery after plastic deformation
- Compressive, shear, and torsional deformations
- Hardness
- Variability of material properties
- Design/safety factors
The photograph of Fig. (a) shows an apparatus that measures the mechanical properties of metals using applied tensile forces. Fig. (b) is a graph (plus insert) that was generated from a tensile test performed by an apparatus such as this on a steel specimen. Data plotted are stress versus strain. The manner in which the mechanical properties of modulus of elasticity (E) as well as yield strength ($\sigma_y$), and tensile strength (TS) are determined as noted on these graphs.
A suspension bridge is shown in Fig. (c). The weight of the bridge deck and automobiles imposes tensile forces on the vertical suspender cables. These forces are in turn transferred to the main suspension cable, which sags in a more-or-less parabolic shape. The metal alloy(s) from which these cables are constructed must meet certain stiffness and strength criteria. Stiffness and strength of the alloy(s) may be assessed from tests performed using a tensile-testing apparatus similar to those shown.
Why Study the Mechanical Properties of Metals?

It is incumbent on engineers to understand how the various mechanical properties are measured and what these properties represent; they may be called upon to design structures/components using predetermined materials such that unacceptable levels of deformation and/or failure will not occur. We demonstrate this procedure with respect to the design of a tensile-testing apparatus in Design Example 6.1.
Learning Objectives

1. Define engineering stress and engineering strain.
2. State Hooke’s law, and note the conditions under which it is valid.
3. Define Poisson’s ratio.
4. Given an engineering stress-strain diagram, determine
   (a) the modulus of elasticity,
   (b) the yield strength (0.002 strain offset),
   (c) the tensile strength
   (d) estimate the percent elongation.
5. For the tensile deformation of a ductile cylindrical specimen, describe changes in specimen profile to the point of fracture.
6. Compute ductility in terms of both percent elongation and percent reduction of area for a material that is loaded in tension to fracture.
7. Give brief definitions of and the units for modulus of resilience and toughness.

8. For a specimen being loaded in tension, given the applied load, the instantaneous cross-sectional dimensions, and original and instantaneous lengths, be able to compute true stress and true strain values.

9. Name the two most common hardness-testing techniques: note two differences between them.

10. (a) Name and briefly describe the two different microindentation hardness testing techniques, and (b) cite situations for which these techniques are generally used.

11. Compute the working stress for a ductile material.
Introduction

- The mechanical behavior of a material reflects the relationship between its response or deformation to an applied load or force.
- **Key mechanical design properties** are **stiffness, strength, hardness, ductility, and toughness.**
- In the United States the most active organization is the American Society for Testing and Materials (ASTM). Its Annual Book of ASTM Standards comprises numerous volumes, which are issued and updated yearly; a large number of these standards relate to mechanical testing techniques.
Factors for the mechanical characteristics of Materials
- The nature of the applied load
- The testing duration
- The environmental conditions

Structural or materials engineers
- The role of structural engineers is to determine stresses and stress distributions within members that are subjected to well-defined loads.
- Materials and metallurgical engineers are concerned with producing and fabricating materials to meet service requirements as predicted by these stress analysis.
Figure 6.1
(a) Schematic illustration of how a tensile load produces an elongation and positive linear strain. Dashed lines represent the shape before deformation; solids line, after deformation.
(b) Schematic illustration of how a compressive load produces contraction and a negative linear strain.
(c) Schematic representation of shear strain $\gamma$, where $\gamma = \tan \theta$.
(d) Schematic representation of torsional deformation (i.e., angle of twist $\phi$) produced by an applied torque $T$. 
• **Tension tests** (Fig. 6.3) (拉伸試驗)
  - **Tension test**: a specimen is deformed, usually to fracture, with a gradually increasing tensile load that is applied *uniaxially* along the long axis of a specimen.
  - A standard tensile specimen is shown in Fig. 6.2. This “dogbone” specimen configuration was chosen so that, during testing, deformation is confined to the narrow center region, and also to reduce the likelihood of fracture at the ends of the specimen. The standard diameter is approximately 12.8 mm, whereas the reduced section length should be at least four times this diameter; 60 mm is common. Gauge length is 50 mm.
  - The specimen is mounted by its ends into the holding grips of the testing apparatus.

*Figure 6.2* A standard tensile specimen with circular cross section.
- The tensile testing machine is designed to elongate the specimen at a constant rate and to continuously and simultaneously measure the instantaneous applied load (with a load cell) and the resulting elongations (using an extensometer).

- A stress-strain test typically takes several minutes to perform and is destructive; that is, the test specimen is permanently deformed and usually fractured. The output of such a tensile test is recorded on a strip chart as load or force versus elongation.

Figure 6.3 Schematic representation of the apparatus used to conduct tensile stress-strain tests. The specimen is elongated by the moving crosshead; load cell and extensometer measure, respectively, the magnitude of the applied load and the elongation.
Tensile testing machine. The force (load) on the sample is recorded on the chart paper in the drawer on the left. The strain that the sample undergoes is also recorded on the chart. The signal for the strain is obtained from the extensometer attached to the sample.
Close-up of the tensile machine extensometer that measures the strain that the sample undergoes during the tensile test. The extensometer is attached to the sample by small spring clamps.
- **Engineering stress** \((\sigma, \text{ N/m}^2)\) (工程應力)

\[
\sigma = \frac{F}{A_0}
\]

\(F\): instantaneous load applied perpendicular to the specimen cross section \((N)\),
\(A_0\): original cross sectional area before any load is applied \((\text{m}^2)\).

- **Engineering strain** \((\varepsilon)\) (工程應變)

\[
\varepsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0}
\]

\(l_i\): instantaneous length \((\text{m})\), \(l_0\): original length \((\text{m})\).
\(\Delta l\): deformation elongation or change in length \((\text{m})\).

Sometimes strain is also expressed as a percentage, in which the strain value is multiplied by 100.
A 1.25-cm-diameter bar is subjected to a load of 2500 kg. Calculate the engineering stress on the bar in megapascals (MPa).

\[ F = ma = 2500kg \times 9.81m/s^2 = 24500N \]

\[ \sigma = \frac{F}{A_0} = \frac{F}{\pi \cdot \left(\frac{d}{2}\right)^2} \]

\[ \Rightarrow \sigma = \frac{24500N}{\pi \cdot \left(\frac{0.0125m}{2}\right)^2} = 2.0 \times 10^8 N/m^2 \]

\[ \Rightarrow \sigma = 200MPa \]
A sample of commercially pure aluminum 0.5 in. wide, 0.4 in. thick, and 8 in. long that has gage markings 2.0 in. apart in the middle of the sample is strained so that the gage markings are 2.65 in. apart. Calculate the engineering strain and the percent engineering strain elongation that the sample undergoes.

\[
\varepsilon = \frac{l - l_0}{l_0}
\]

\[
\Rightarrow \varepsilon = \frac{2.65\text{ in.} - 2.0\text{ in.}}{2.0\text{ in.}}
\]

\[
\Rightarrow \varepsilon = 0.325
\]

\[
\Rightarrow \varepsilon = 32.5\%
\]
**Compression tests** (壓縮試驗)

- A compression test is conducted in a manner similar to the tensile test, except that the force is compressive and the specimen contracts along the direction of the stress.

- By convention, a compressive force is taken to be negative, which yields a negative stress. Furthermore, because \( l_0 \) is greater than \( l_i \), compressive strains are also negative.

\[
\sigma = \frac{F}{A_0}
\]

\[
\varepsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0}
\]
• **Shear tests** (剪切試驗)
  - **Shear stress** ($\tau$) (剪應力)
    \[
    \tau = \frac{F}{A_0}
    \]
    $F$: load or force imposed parallel to the upper and lower faces, each of which has an area of $A_0$.
  - **Shear strain** ($\gamma$) (剪應變): defined as the tangent of the strain angle $\theta$.
    \[
    \gamma = \frac{d}{l} = \tan \theta
    \]
  - **Shear modulus** ($G$) (剪力模數)
    \[
    \tau = G\gamma
    \]
- Geometric considerations of the stress state
  - Consider the cylindrical tensile specimen (Fig. 6.4) that is subjected to a tensile stress $\sigma$ applied to its axis. Furthermore, consider also the plane $p-p'$ that is oriented at some arbitrary angle $\theta$ relative to the plane of the specimen end-face.
  - Upon this plane $p-p'$, the applied stress is no longer a pure tensile one. Rather, a more complex stress state is present that consists of a tensile (or normal) stress $\sigma'$ that acts normal to the $p-p'$ plane and, in addition, a shear stress $\tau'$ that acts parallel to this plane; both of these stresses are represented in the figure.

**Figure 6.4** Schematic representation showing normal ($\sigma'$) and shear ($\tau'$) stresses that act on a plane oriented at an angle $\theta$ relative to the plane taken perpendicular to the direction along which a pure tensile stress ($\sigma$) is applied.
**Concepts of Stress and Strain**

\[
\cos \theta = \frac{A_0}{A_\theta} \Rightarrow A_\theta = \frac{A_0}{\cos \theta}
\]

\[
F = \sigma \cdot A_0
\]

\[
F' = F \cdot \cos \theta = \sigma \cdot A_0 \cdot \cos \theta
\]

\[
F'' = F \cdot \sin \theta = \sigma \cdot A_0 \cdot \sin \theta
\]

\[
F' = \sigma' \cdot A_\theta = \frac{\sigma' \cdot A_0}{\cos \theta}
\]

\[
\Rightarrow \frac{\sigma' \cdot A_0}{\cos \theta} = \sigma \cdot A_0 \cdot \cos \theta \Rightarrow \sigma' = \sigma \cdot \cos^2 \theta = \sigma \cdot \left( \frac{1 + \cos 2\theta}{2} \right)
\]

\[
F'' = \tau' \cdot A_\theta = \frac{\tau' \cdot A_0}{\cos \theta}
\]

\[
\Rightarrow \frac{\tau' \cdot A_0}{\cos \theta} = \sigma \cdot A_0 \cdot \sin \theta \Rightarrow \tau' = \sigma \cdot \sin \theta \cdot \cos \theta = \sigma \cdot \left( \frac{\sin 2\theta}{2} \right)
\]
Hooke’s law (虎克定律)
- For most metals that are stressed in tension and at relatively low levels, stress and strain are proportional to each other through the relationship

\[ \sigma = E\varepsilon \]

\( E \): modulus of elasticity (弾性模數) or Young’s modulus (楊氏模數), GPa. This is known as Hook’s law.

Table 6.1 Room-temperature elastic and shear moduli, and Poisson’s ratio for various metal alloys.

<table>
<thead>
<tr>
<th>Metal alloy</th>
<th>Modulus of elasticity (GPa)</th>
<th>Shear modulus (GPa)</th>
<th>Poisson’s ratio (包松比)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>69</td>
<td>25</td>
<td>0.33</td>
</tr>
<tr>
<td>Brass</td>
<td>97</td>
<td>37</td>
<td>0.34</td>
</tr>
<tr>
<td>Copper</td>
<td>110</td>
<td>46</td>
<td>0.34</td>
</tr>
<tr>
<td>Magnesium</td>
<td>45</td>
<td>17</td>
<td>0.29</td>
</tr>
<tr>
<td>Nickel</td>
<td>207</td>
<td>76</td>
<td>0.31</td>
</tr>
<tr>
<td>Steel</td>
<td>207</td>
<td>83</td>
<td>0.30</td>
</tr>
<tr>
<td>Titanium</td>
<td>107</td>
<td>45</td>
<td>0.34</td>
</tr>
<tr>
<td>Tungsten</td>
<td>407</td>
<td>160</td>
<td>0.28</td>
</tr>
</tbody>
</table>
Interactive MSE
- Tensile tests
- Tensile tests on metal alloys
- Titanium, temp. steel, aluminum, carbon steel, cast iron
**Elastic deformation (彈性變形)**
- Deformation in which stress and strain are proportional is called elastic deformation.
- Elastic deformation is nonpermanent, which means that when the applied load is released, the piece returns to its original shape (Fig. 6.5).

**Modulus of elasticity (弾性模數)**
- In Fig. 6.5, the slope of this linear segment corresponds to the modulus of elasticity \( E \).
- This modulus may be thought of as stiffness, or a material’s resistance to elastic deformation.

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**Figure 6.5** Schematic stress-strain diagram showing linear elastic deformation for loading and unloading cycles.
• **Tangent or secant modulus**
  - There are some materials for which this elastic portion of the stress-strain curve is not linear (Fig. 6.6). For this nonlinear behavior, either tangent or secant modulus is normally used.
  - Tangent modulus is taken as the slope of the stress-strain curve at some specified level of stress, whereas secant modulus represents the slope of a secant drawn from the origin to some given point of the $\sigma$-$\varepsilon$ curve.

Figure 6.6 Schematic stress-strain diagram showing nonlinear elastic behavior, and how secant and tangent moduli are determined.
• **Modulus of elasticity**
  - The magnitude of the modulus of elasticity is a measure of the resistance to separation of adjacent atoms, that is, the interatomic bonding force.
  - This modulus is proportional to the slope of the interatomic force-separation curve at the equilibrium spacing:

\[
E = \frac{d\sigma}{d\varepsilon} = \frac{A_0}{r_0} \propto \left( \frac{dF}{dr} \right)_{r_0}
\]

**Figure 6.7** Force versus interatomic separation for weakly and strongly bonded atoms. The magnitude of the modulus of elasticity is proportional to the slope of each curve at the equilibrium interatomic separation \(r_0\).
Values of the modulus of elasticity for ceramic materials are about the same as for metals; for polymers they are lower (Fig. 1.4). These differences are a direct consequence of the different types of atomic bonding in the three materials types.

With increasing temperature, the modulus of elasticity diminishes (Fig. 6.8).

Figure 6.8 Plot of modulus of elasticity versus temperature for tungsten, steel, and aluminum.
**Anelasticity** (滯彈性，黏彈性)

- In most engineering materials, there will exist a time-dependent elastic strain component.
- Elastic deformation will continue after the stress application, and upon load release some finite time is required for complete recovery. This **time-dependent elastic behavior is known as anelasticity**.
- For metals, the anelastic component is normally small and is often neglected. However, for some polymeric materials its magnitude is significant; in this case it is termed **viscoelastic (黏彈性的) behavior**.
黏彈性之Maxwell模型係由彈簧與緩衝筒串聯形成。

當黏彈性固體受到應力$\sigma_c$作用
弹簧首先會產生一瞬間應變$\varepsilon_1$
$$\varepsilon_1 = \frac{\sigma_c}{E}$$
同時緩衝筒亦將產生一應變$\varepsilon$
$$\frac{d\varepsilon}{dt} = \frac{\sigma_c}{\eta}$$
經過某時間$t_1$後之總應變$\varepsilon_t$
$$\varepsilon_t = \sigma_c \cdot \left(\frac{1}{E} + \frac{t_1}{\eta}\right)$$
如果應變維持至$t_2$不變$(d\varepsilon/dt = 0)$
則應力鬆弛現象將會發生$(\sigma_c \rightarrow \sigma_1)$
$$\frac{d\varepsilon}{dt} = \left(\frac{1}{E}\right) \cdot (d\sigma / dt) + \sigma \cdot \left(\frac{1}{\eta}\right)$$
$$\Rightarrow \sigma_t = \sigma_c \cdot \exp(-Et / \eta)$$
如果在$t_2$時將應力$\sigma_1$移除，將會產生一彈性應變回復$\varepsilon_2$
$$\varepsilon_2 = \frac{\sigma_1}{E}$$
黏彈性之Voigt-Kelvin模型係由弾簧與緩衝筒並聯形成。

time: 0 → t₁

當黏彈性固體受到應力σ作用，

應力將由彈簧σₛ和緩衝筒σ₃共同分擔

\[ \sigma = \sigma_s + \sigma_d \]

⇒ \[ \sigma = E \varepsilon + \eta \cdot (d \varepsilon / dt) \]

⇒ \[ dt / \eta = d \varepsilon / (\sigma - E \varepsilon) \]

⇒ \[ t / \eta = -(1/E) \cdot Ln(\sigma - E \varepsilon) \]

⇒ \[ \varepsilon = (1/E) \cdot [\sigma - \exp(-Et/\eta)] \]

Voigt-Kelvin模型對熱塑性材料之潛變行為或回復行為，有相當程度的回應，但欠缺應力鬆弛現象。

時間: t₂ →

如果將應力除去，將會有應變回覆發生

\[ \sigma = \sigma_s + \sigma_d = 0 \]

⇒ \[ 0 = E \varepsilon + \eta \cdot (d \varepsilon / dt) \]

⇒ \[ d \varepsilon / \varepsilon = -(E / \eta) \cdot dt \]

⇒ \[ \varepsilon_t = \varepsilon_0 \cdot \exp(-Et/\eta) \]
A piece of copper originally 305 mm long is pulled in tension with a stress 276 MPa. If the deformation is entirely elastic, what will be the resultant elongation? 

\( E_{Cu} = 110 \, \text{GPa} \)

\[
\sigma = E \varepsilon = E \cdot \frac{\Delta l}{l_0} \implies \Delta l = \frac{\sigma \cdot l_0}{E}
\]

\[
\Delta l = \frac{276 \times 10^6 \, \text{Pa} \times 305 \, \text{mm}}{110 \times 10^9 \, \text{Pa}} = 0.765 \, (\text{mm})
\]
**Poisson’s ratio (包松比)**

- When a tensile stress is imposed on a metal specimen, an elastic elongation and accompanying strain $\varepsilon_Z$ result in the direction of the applied stress (Fig. 6.9). As a result of this elongation, there will be constrictions in the lateral ($x$ & $y$) directions perpendicular to the applied stress.

- If the applied stress is uniaxial (only in the $z$ direction), and the material is isotropic, then $\varepsilon_x = \varepsilon_y$.

- Poisson’s ratio $\nu$ is defined as the ratio of the lateral and axial strains.

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z}$$

**Figure 6.9** Axial ($z$) elongation (positive strain) and lateral ($x$ and $y$) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.
The negative sign is included in the expression so that $\nu$ will always be positive, since $\varepsilon_x$ and $\varepsilon_y$ will always be of opposite sign.

- Poisson’s ratio for isotropic materials should be $\frac{1}{4}$.
- The maximum value for $\nu$ (no net volume change) is $\frac{1}{2}$.
- For many metals and other alloys, values of Poisson’s ratio between 0.25 and 0.35.

**Relationship among moduli**

- For isotropic materials, shear and elastic moduli are related to each other and to Poisson ratio according to
  
  $$E = 2G \cdot (1 + \nu)$$

- In most metals $G$ is about $0.4E$; thus, if the value of one modulus is known, the other may be approximated.

**Elastic anisotropy**

- Many materials are elactically anisotropic; that is, the elastic behavior varies with crystallographic direction.
- Because the grain orientation is random in most polycrystalline materials, these may be considered to be isotropic.
A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm. Determine the magnitude of the load required to produce a $2.5 \times 10^{-3}$ mm change in diameter if the deformation is entirely elastic. (Poisson’s ratio: 0.34, $E$: 97 GPa)

\[ \varepsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3} \text{mm}}{10 \text{mm}} = -2.5 \times 10^{-4} \]

\[ \varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{-2.5 \times 10^{-4}}{0.34} = 7.35 \times 10^{-4} \]

\[ \sigma = E\varepsilon_z = \left(97 \times 10^3 \text{MPa}\right) \times \left(7.35 \times 10^{-4}\right) \]

\[ \Rightarrow \sigma = 71.3 \text{MPa} \]

\[ F = \sigma A_0 = \sigma \cdot \pi \cdot \left(\frac{d_0}{2}\right)^2 \]

\[ \Rightarrow F = \left(71.3 \times 10^6 \text{N/m}^2\right) \times \pi \times \left(\frac{10 \times 10^{-3} \text{m}}{2}\right)^2 \]

\[ \Rightarrow F = 5600 \text{N} \]
- **Plastic deformation** (塑性変形)
  - A permanent, nonrecoverable deformation.
  - From an atomic perspective, plastic deformation corresponds to the breaking of bonds with original atom neighbors and then re-forming bonds with new neighbors as large numbers of atoms or molecules move relative to one another; upon removable of the stress they do not return to their original positions.
  - For crystalline solids, deformation is accomplished by means of a process called slip, which involves the motion of dislocations. Plastic deformation in noncrystalline solids occurs by a viscous flow mechanism.

- **Comparison**

![Diagram showing the difference between elastic and plastic deformation](image-url)
Tensile Properties

- **Yielding** (降伏)
The stress level at which plastic deformation begins.

- **Proportional limit** (比例限)
For metals that experience this gradual elastic-plastic transition, the point of yielding may be determined as the initial departure from linearity of the stress-strain curve, as indicated by point $P$.

- **Yield strength $\sigma_y$** (降伏強度)
  - A convention has been established wherein a straight line is constructed parallel to the elastic portion of the stress-strain curve at some specified strain offset, usually 0.002. The stress corresponding to the intersection of this line and the stress-strain curve as it bends over in the plastic region is defined as the yield strength.

**Figure 6.10(a)** Typical stress-strain behavior for a metal showing elastic and plastic deformations, the proportional limit $P$, and the yield strength $\sigma_y$, as determined using the 0.002 strain offset method.
- The magnitude of the yield strength for a metal is a measure of its resistance to plastic deformation.

- **Yield point (降伏點)**
  - The elastic-plastic transition of some steels is well defined and occurs abruptly in what is termed a yield point phenomenon. At the upper yield point, plastic deformation is initiated with an actual decrease in stress. Continued deformation fluctuates slightly about some constant stress value, termed the lower yield point; stress subsequently rises with increasing strain.
  - For metals that display this effect, the yielding strength is taken as the average stress that is associated with the lower yield point, because it is well defined and relative to the testing procedure.

**Figure 6.10 (b)** Representative stress-strain behavior found for some steels demonstrating the yield point phenomenon.
- **Tensile strength** (抗拉強度)

  - The tensile strength $TS$ (MPa) is the stress at the maximum on the engineering stress-strain curve (Fig. 6.11). This corresponds to the maximum stress that can be sustained by a structure in tension; if this stress is applied and maintained, fracture will result. All deformation up to this point is uniform throughout the narrow region of the tensile specimen.

  - However, at this maximum stress, a small constriction or neck (頸縮) begins to form at some point, and all subsequent deformation is confined at this neck.

**Figure 6.11** Typical engineering stress-strain behavior to fracture, point $F$. The tensile strength $TS$ in indicated at point $M$. The circular insets represent the geometry of the deformed specimen at various points along the curve.
This phenomenon is termed “necking,” and fracture ultimately occurs at the neck.
- The fracture strength corresponds to the stress at fracture.
- Tensile strengths may vary anywhere from 50 MPa for an aluminum to as high as 3000 MPa for the high-strength steels.
- Ordinarily, when the strength of a metal is cited for design purposes, the yield strength is used.
Fracture strengths are not normally specified for engineering design purposes.
## Tensile Properties

### Table 6.2  Typical Mechanical Properties of Several Metals and Alloys in an Annealed State

<table>
<thead>
<tr>
<th>Metal Alloy</th>
<th>Yield Strength, MPa (ksi)</th>
<th>Tensile Strength, MPa (ksi)</th>
<th>Ductility, %EL [in 50 mm (2 in.)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>35 (5)</td>
<td>90 (13)</td>
<td>40</td>
</tr>
<tr>
<td>Copper</td>
<td>69 (10)</td>
<td>200 (29)</td>
<td>45</td>
</tr>
<tr>
<td>Brass (70Cu–30Zn)</td>
<td>75 (11)</td>
<td>300 (44)</td>
<td>68</td>
</tr>
<tr>
<td>Iron</td>
<td>130 (19)</td>
<td>262 (38)</td>
<td>45</td>
</tr>
<tr>
<td>Nickel</td>
<td>138 (20)</td>
<td>480 (70)</td>
<td>40</td>
</tr>
<tr>
<td>Steel (1020)</td>
<td>180 (26)</td>
<td>380 (55)</td>
<td>25</td>
</tr>
<tr>
<td>Titanium</td>
<td>450 (65)</td>
<td>520 (75)</td>
<td>25</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>565 (82)</td>
<td>655 (95)</td>
<td>35</td>
</tr>
</tbody>
</table>
From the tensile-strain behavior for the brass specimen shown in Figure 6.12, determine the following:

(a) The modulus of elasticity.
(b) The yield strength at a strain offset of 0.002.
(c) The maximum load that can be sustained by a cylindrical specimen having an original diameter of 12.8 mm.
(d) The change in length of a specimen of originally 250 mm long that is subjected to a tensile stress of 345 MPa.

Figure 6.12 The stress-strain behavior for the brass specimen.
Example Problem 6.3

(a) \[ E = \text{slope} = \frac{\Delta \sigma}{\Delta \varepsilon} \]

\[ \Rightarrow E = \frac{(150 - 0) \text{MPa}}{0.0016 - 0} = 93.8 \text{GPa} \]

(b) Intersection is \(~250\ \text{MPa}\).

(c) \[ F = \sigma A_0 = \sigma \cdot \pi \cdot \left(\frac{d_0}{2}\right)^2 \]

\[ \Rightarrow F = \left(450 \times 10^6 \text{N/m}^2\right) \times \pi \times \left(\frac{12.8 \times 10^{-3} \text{m}}{2}\right)^2 \]

\[ \Rightarrow F = 57900 \text{N} \]

(d) \[ d = \varepsilon_0 l = 0.06 \times 250 \text{mm} = 15 \text{mm} \]
**Ductility** (延性)

- Ductility is a measure of the degree of plastic deformation that has been sustained at fracture. Ductility may be expressed quantitatively as either percent elongation or percent reduction in area.

- The percent elongation %EL is the percentage of plastic strain at fracture.

\[
%\text{EL} = \left( \frac{l_f - l_0}{l_0} \right) \times 100
\]

\(l_f\): fracture length, \(l_0\): original gauge length.

The shorter \(l_0\), the greater the fraction of total elongation from the neck and the higher value of %EL.

Therefore, \(l_0\) should be specified when percent elongation values are cited; it is commonly 50 mm.

**Figure 6.13** Schematic representations of tensile stress-strain behavior for brittle and ductile metals loaded to fracture.
- Percent reduction in area $\%RA$ is defined as

$$\%RA = \left( \frac{A_0 - A_f}{A_0} \right) \times 100$$

$A_0$: original cross-sectional area, $A_f$: cross-sectional area at the point of fracture.

Percent reduction in area values are independent of both $l_0$ and $A_0$.

- A knowledge of the ductility of materials is important for at least two reasons.
  1. It indicates to a designer the degree to which a structure will deform plastically before fracture.
  2. It specifies the degree of allowable deformation during fabrication operations.

- Brittle materials are approximately considered to be those having a fracture strain of less than about 5\%.
- As with modulus of elasticity, the magnitudes of both yield and tensile strengths decline with increasing temperature; just the reverse holds for ductility – it usually increases with temperature.

Figure 6.14 Engineering stress-strain behavior for iron at three temperatures.
A 0.5-in.-diameter round sample of a 1030 carbon steel is pulled to failure in a tensile testing machine. The diameter of the sample was 0.343 in. at the fracture surface. Calculate the percent reduction in area of the sample.

\[ \% RA = \frac{A_0 - A_f}{A_0} \times 100\% \]

\[ \Rightarrow \% RA = \frac{\pi \left( \frac{0.5\text{in.}}{2} \right)^2 - \pi \left( \frac{0.343\text{in.}}{2} \right)^2}{\pi \left( \frac{0.5\text{in.}}{2} \right)^2} \times 100\% \]

\[ \Rightarrow \% RA = 53\% \]
**Resilience** (彈性能)
- Resilience is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered.
- The modulus of resilience, $U_r$ (J/m$^3$), is the strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding. The modulus of resilience for a specimen subjected to a uniaxial tension test is just the area under the engineering stress-strain curve taken to yielding, or

$$U_r = \int_0^{\varepsilon_y} \sigma d\varepsilon$$

Assuming a linear elastic region.

$$U_r = \frac{1}{2} \sigma_y \varepsilon_y = \frac{1}{2} \sigma_y \left(\frac{\sigma_y}{E}\right) = \frac{\sigma_y^2}{2E}$$

$\varepsilon_y$: strain at yielding.

- Resilient materials are those having high yield strengths and low moduli of elasticity; such alloys would be used in spring applications.

**Figure 6.15** Schematic representation showing how modulus of resilience is determined from the tensile stress-strain behavior of a material.
- **Toughness** (J/m$^3$) (韌性，韌度)
  - Toughness is the ability of a material to absorb energy and plastically deform before fracturing.
  - For dynamic (high strain rate) loading conditions and when a notch (or point of stress concentration) is present, notch toughness is assessed by using an impact test.
  - Fracture toughness is a property indicative of a material’s resistance to fracture when a crack is present. Fracture toughness is a major consideration for all structural materials.
  - For the static (low strain rate) situation, toughness may be ascertained from the results of a tensile-strain test. It is the area under $\sigma$-$\varepsilon$ curve up to the point of fracture.
  - For a metal to be tough, it must display both strength and ductility.
Figure 8.11
(a) Specimen used for Charpy and Izod impact tests.
(b) A schematic drawing of an impact testing apparatus. The chamber is released from fixed height $h$ and strikes the specimen; the energy expended in fracture is reflected in the difference between $h$ and the swing height $h'$. Specimen placements for both Charpy and Izod tests are also shown.
### Tensile Properties

#### Table 6.3 Tensile Stress–Strain Data for Several Hypothetical Metals to Be Used with Concept Checks 6.2 and 6.4

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Strength (MPa)</th>
<th>Tensile Strength (MPa)</th>
<th>Strain at Fracture</th>
<th>Fracture Strength (MPa)</th>
<th>Elastic Modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>310</td>
<td>340</td>
<td>0.23</td>
<td>265</td>
<td>210</td>
</tr>
<tr>
<td>B</td>
<td>100</td>
<td>120</td>
<td>0.40</td>
<td>105</td>
<td>150</td>
</tr>
<tr>
<td>C</td>
<td>415</td>
<td>550</td>
<td>0.15</td>
<td>500</td>
<td>310</td>
</tr>
<tr>
<td>D</td>
<td>700</td>
<td>850</td>
<td>0.14</td>
<td>720</td>
<td>210</td>
</tr>
<tr>
<td>E</td>
<td>Fractures before yielding</td>
<td>650</td>
<td></td>
<td></td>
<td>350</td>
</tr>
</tbody>
</table>
**True Stress and Strain**

**Introduction**
- From Fig. 6.11, the decline in the stress necessary to continue deformation past the maximum, point $M$, seems to indicate that the metal is becoming weaker.
- This is not at all the case; as a matter of fact, it is increasing in strength. (necking)

**True stress** $\sigma_T$ (真應力) and **True strain** $\varepsilon_T$ (真應變)

- True stress $\sigma_T$ is defined as the load $F$ divided by the instantaneous cross-sectional area $A_i$ over which deformation is occurring.

$$\sigma_T = \frac{F}{A_i} \quad \varepsilon_T = \int_{l_0}^{l_i} \frac{dl}{l} = \ln \frac{l_i}{l_0} = \frac{l_i - l_0}{l_0}$$

$F$: load, $A_i$: instantaneous cross-sectional area, $l_i$: instantaneous length, $l_0$: original length.

**Figure 6.16** A comparison of typical tensile engineering stress-strain and true stress-strain behaviors. Necking begins at point $M$ on the engineering curve, which corresponds to $M'$ on the true curve. The “corrected” true stress-strain curve takes into account the complex stress state within the neck region.
True Stress and Strain

- The relationship between true strain $\varepsilon_T$ and engineering strain $\varepsilon$

$$\varepsilon = \frac{l_i - l_0}{l_0} = \frac{l_i}{l_0} - 1$$

$$\Rightarrow \frac{l_i}{l_0} = 1 + \varepsilon$$

$$\Theta \varepsilon_T = \ln \frac{l_i}{l_0}$$

$$\Rightarrow \varepsilon_T = \ln (1 + \varepsilon)$$

- The relationship between true stress $\sigma_T$ and engineering stress $\sigma$

If no volume change $\Rightarrow A_i l_i = A_0 l_0$

$$\sigma_T = \frac{F}{A_i} = \frac{F}{A_i} \cdot \frac{A_0}{A_0} = \frac{F}{A_0} \cdot \frac{l_i}{l_0} = \sigma \cdot \frac{l_i}{l_0}$$

$$\Theta \frac{l_i}{l_0} = 1 + \varepsilon$$

$$\therefore \sigma_T = \sigma \cdot (1 + \varepsilon)$$
Comparison of the true stress-true strain curve with the engineering (nominal) stress-strain diagram for a low-carbon steel.
- Coincident with the formation of a neck is the introduction of a complex stress state within the neck region (i.e., the existence of other stress components in addition to the axial stress).

As a consequence, the correct stress (axial) within the neck is slightly lower than the stress computed from the applied load and neck cross-sectional area.

- For some metals and alloys the region of the true stress-strain curve to the point at which necking begins may be approximated by \( \sigma_T = k\varepsilon_T^n \).  
  \( (k \text{ & } n: \text{constants}) \)

The parameter \( n \) is often termed the strain-hardening exponent and has a value less than unity.
A cylindrical specimen of steel having an original diameter of 12.8 mm is tensile-tested to fracture and found to have an engineering fracture strength $\sigma_f$ of 460 MPa. If its cross-sectional diameter at fracture is 10.7 mm, determine: (a) The ductility in terms of percent reduction in area. (b) The true stress at fracture.

(a) $\% RA = \frac{A_0 - A_f}{A_0} \times 100\%$

$\Rightarrow \% RA = \frac{\pi \cdot \left(\frac{12.8mm}{2}\right)^2 - \pi \cdot \left(\frac{10.7mm}{2}\right)^2}{\pi \cdot \left(\frac{12.8mm}{2}\right)^2} \times 100\% = 30\%$

(b) $F = \sigma_f A_0$

$\Rightarrow F = \left(460 \times 10^6 \text{ N/m}^2\right) \cdot \pi \cdot \left(\frac{12.8mm}{2}\right)^2 \cdot \left(\frac{1m^2}{10^6 \text{mm}^2}\right) = 59200 \text{ N}$

$\sigma_T = \frac{F}{A_f} = \frac{59200 \text{ N}}{\pi \cdot \left(\frac{10.7mm}{2}\right)^2 \cdot \left(\frac{1m^2}{10^6 \text{mm}^2}\right)} = 6.6 \times 10^8 \text{ N/m}^2 = 660 \text{ MPa}$
Compute the strain-hardening exponent $n$ for an alloy in which a true stress of 415 MPa produces a true strain of 0.10; assume a value of 1035 MPa for $k$.

\[
\sigma_T = k\varepsilon_T^n \\
\implies \log \sigma_T = \log k + n \cdot \log \varepsilon_T \\
\implies n = \frac{\log \sigma_T - \log k}{\log \varepsilon_T} \\
\implies n = \frac{\log(415 \times 10^6 N/m^2) - \log(1035 \times 10^6 N/m^2)}{\log(0.1)} = 0.397
\]
- Upon release of the load during the course of a stress-strain test, some fraction of the total deformation is recovered as elastic strain (Fig. 6.17).

During the unloading cycle, the curve traces a near straight-line path from the point of unloading (point $D$), and its slope is virtually identical to the modulus of elasticity, or parallel to the initial elastic portion of the curve. If the load is reapplied, the curve will traverse essentially the same linear portion in the direction opposite to unloading; yielding will again occur at the unloading stress level where the unloading began.

**Figure 6.17** Schematic tensile stress-strain diagram showing the phenomena of elastic strain recovery and strain hardening. The initial yield strength is designated as $\sigma_{y0}$; $\sigma_{yi}$ is the yield strength after releasing the load at point $D$, and then upon reloading.
Hardness (硬度)

- Hardness is a measure of a material’s resistance to localized plastic deformation.
- Early hardness tests were based on natural minerals with a scale constructed solely on the ability of one material to scratch another that was softer. Mohs scale ranged from 1 on the soft end for talc to 10 for diamond.
  1. talc (滑石)
  2. gypsum (石膏)
  3. calcite (方解石)
  4. fluorite (萤石)
  5. apatite (磷灰石)
  6. feldspar (长石)
  7. quartz (石英)
  8. beryl (绿宝石)
  9. corundum (金刚砂)
  10. diamond (鑽石)
- For quantitative hardness techniques, a small indenter is forced into the surface of a material to be tested, under controlled conditions of load and rate of application. The depth or size of the resulting indentation is measured, which in turn is related to a hardness number; the softer the material, the larger and deeper the indentation, and the lower the hardness index number.

- Hardness tests are performed more frequently for several reasons:
  1. They are simple and inexpensive—ordinarily no special specimen need be prepared, and the testing apparatus is relatively inexpensive.
  2. The test is nondestructive—the specimen is neither fractured nor excessively deformed; a small indentation is the only deformation.
  3. Other mechanical properties often may be estimated from hardness data, such as tensile strength.
# Hardness

## Table 6.5 Hardness-Testing Techniques

<table>
<thead>
<tr>
<th>Test</th>
<th>Indenter</th>
<th>Shape of indentation</th>
<th>Load</th>
<th>Formula for hardness number</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Brinell</strong></td>
<td>10-mm sphere of steel or WC</td>
<td><img src="image" alt="Brinell Indenter" /></td>
<td>$P$</td>
<td>$\text{HB} = \frac{2P}{\pi D [D^2 - d^2]}$</td>
</tr>
<tr>
<td><strong>Vickers microhardness</strong></td>
<td>Diamond pyramid</td>
<td><img src="image" alt="Vickers Indenter" /></td>
<td>$P$</td>
<td>$\text{HV} = 1.854 \frac{P}{d_1^2}$</td>
</tr>
<tr>
<td><strong>Knoop microhardness</strong></td>
<td>Diamond pyramid</td>
<td><img src="image" alt="Knoop Indenter" /></td>
<td>$P$</td>
<td>$\text{HK} = 1.42 \frac{P}{l^2}$</td>
</tr>
<tr>
<td><strong>Rockwell &amp; superficial Rockwell</strong></td>
<td>Diamond cone 1/16”, 1/8”, 1/4”, 1/2” diameter Steel spheres</td>
<td><img src="image" alt="Rockwell Indenter" /></td>
<td>Rockwell 60, 100, 150 Kg</td>
<td>Superficial Rockwell 15, 30, 45 Kg</td>
</tr>
</tbody>
</table>
(b) Steps in the measurement of hardness with a diamond-cone indenter. The depth \( t \) determines the hardness of the material. The lower the value of \( t \), the harder the material.
• **Rockwell hardness tests**
  - The Rockwell tests constitute the most common method used to measure hardness.
  - Several different scales may be utilized from possible combinations of various indenters and different loads, which permit the testing of virtually all metal alloys.
  - Indenters include spherical and hardened steel balls having diameters of 1.558, 3.175, 6.350, and 12.70 mm, and a conical diamond (Brale) indenter, which is used for the hardest materials.
  - With this system, a hardness number is determined by the difference in depth of penetration resulting from the application of an initial minor load (Rockwell: 10Kg, superficial Rockwell: 3 Kg) followed by a larger major load (Rockwell: 60 kg, 100 kg, 150 kg, superficial Rockwell: 15 kg, 30 kg, 45 kg); utilization of a minor load enhances test accuracy (20-100).
  - When specifying Rockwell and superficial hardesses, both hardness number and scale symbol must be indicated. For example, 80 HRB represents a Rockwell hardness of 80 on the B scale, and 60 HR30W indicates a superficial hardness of 60 on the 30W scale.
- Specimen thickness should be at least ten times the indentation depth, whereas allowance should be made for at least three indentation diameters between the center of one indentation and the specimen edge, or to the center of a second indentation.
- Accuracy is dependent on the indentation being made into a smooth flat surface.
## Hardness

### Table 6.6a  Rockwell Hardness Scales

<table>
<thead>
<tr>
<th>Scale Symbol</th>
<th>Indenter</th>
<th>Major Load (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Diamond</td>
<td>60</td>
</tr>
<tr>
<td>B</td>
<td>( \frac{1}{16} ) - in. ball</td>
<td>100</td>
</tr>
<tr>
<td>C</td>
<td>Diamond</td>
<td>150</td>
</tr>
<tr>
<td>D</td>
<td>Diamond</td>
<td>100</td>
</tr>
<tr>
<td>E</td>
<td>( \frac{1}{8} ) - in. ball</td>
<td>100</td>
</tr>
<tr>
<td>F</td>
<td>( \frac{1}{16} ) - in. ball</td>
<td>60</td>
</tr>
<tr>
<td>G</td>
<td>( \frac{1}{8} ) - in. ball</td>
<td>150</td>
</tr>
<tr>
<td>H</td>
<td>( \frac{1}{16} ) - in. ball</td>
<td>60</td>
</tr>
<tr>
<td>K</td>
<td>( \frac{1}{8} ) - in. ball</td>
<td>150</td>
</tr>
</tbody>
</table>

### Table 6.6b  Superficial Rockwell Hardness Scales

<table>
<thead>
<tr>
<th>Scale Symbol</th>
<th>Indenter</th>
<th>Major Load (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15N</td>
<td>Diamond</td>
<td>15</td>
</tr>
<tr>
<td>30N</td>
<td>Diamond</td>
<td>30</td>
</tr>
<tr>
<td>45N</td>
<td>Diamond</td>
<td>45</td>
</tr>
<tr>
<td>15T</td>
<td>( \frac{1}{16} ) - in. ball</td>
<td>15</td>
</tr>
<tr>
<td>30T</td>
<td>( \frac{1}{16} ) - in. ball</td>
<td>30</td>
</tr>
<tr>
<td>45T</td>
<td>( \frac{1}{8} ) - in. ball</td>
<td>45</td>
</tr>
<tr>
<td>15W</td>
<td>( \frac{1}{8} ) - in. ball</td>
<td>15</td>
</tr>
<tr>
<td>30W</td>
<td>( \frac{1}{8} ) - in. ball</td>
<td>30</td>
</tr>
<tr>
<td>45W</td>
<td>( \frac{1}{8} ) - in. ball</td>
<td>45</td>
</tr>
</tbody>
</table>
• Brinell hardness tests
  - In Brinell tests, a hard, spherical indenter is forced into the surface of the metal to be tested. The diameter of the hardened steel (or WC) indenter is 10.00 mm. Standard loads range between 500 and 3000 kg in 500-kg increments; during a test, the load is maintained constant for a specified time (between 10 and 30 s).
  - The Brinell hardness number, HB, is a function of both the magnitude of the load and the diameter of the resulting indentation. This diameter is measured with a special low-power microscope, utilizing a scale that is etched on the eyepiece. The measured diameter is then converted to the appropriate HB number using a chart.
  - Maximum specimen thickness as well as indentation position (relative to specimen edges) and minimum indentation spacing requirements are the same as for Rockwell tests.
Knoop and Vickers microhardness tests (微硬度測試)

- For each test a very small diamond indenter having pyramidal geometry is forced into the surface of the specimen. Applied loads are ranging between 1 and 1000 g. The resulting impression is observed under a microscope and measured; this measurement is then converted into a hardness number.
- Careful specimen surface preparation (grinding and polishing) may be necessary to ensure a well-defined indentation that may be accurately measured.
- The Knoop and Vickers hardness numbers are designated by HK and HV, respectively, and the hardness scales for both techniques are approximately equivalent.
- Knoop and Vickers are referred to as microindentation-testing methods on the basis of indenter size. Both are well suited for measuring the hardness of small, selected specimen regions; furthermore, Knoop is used for testing brittle materials such as ceramics.
• **Hardness conversion**
  - Because hardness is not a well-defined material property, and because of the experimental dissimilarities among the various techniques, a comprehensive conversion scheme has not been devised. Hardness conversion data have been determined experimentally and found to be dependent on material type and characteristics.
  - The most reliable conversion data exist for steels, some of which are presented in Fig. 6.18 for Knoop, Brinell, and two Rockwell scales.

**Figure 6.18** Comparison of several hardness scales.
Correlation between hardness and tensile strength
- Both tensile strength and hardness are indicators of a metal’s resistance to plastic deformation. Consequently, they are roughly proportional, as Fig. 6.19 indicates.
- As a rule of thumb for most steels, the HB and the tensile strength are related according to

\[
TS(MPa) = 3.45 \times HB \\
TS(psi) = 500 \times HB
\]

Figure 6.19 Relationships between hardness and tensile strength for steel, brass, and cast iron.
Introduction

- Even if we have a most precise measuring apparatus and a highly controlled test procedure, there will always be some scatter or variability in the data that are collected from specimens of the same material.

- A number of factors lead to uncertainties in measured data. These include the test method, variations in specimen fabrication procedures, operator bias, and apparatus calibration. Furthermore, inhomogeneities may exist within the same lot of material, and/or slight compositional and other differences from lot to lot.

- It is important for the designer to realize that scatter and variability of materials properties are inevitable and must be dealt with appropriately.

“What is the fracture strength of this alloy?” ⇒ “What is the probability of failure of this alloy under these given circumstances?”

- It is often desirable to specify a typical value and degree of dispersion (or scatter) for some measured property.
Variability of Material Properties

- Computation of average and standard deviation values

- An average value is obtained by dividing the sum of all measured values by the number of measurements taken.

\[
\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}
\]

\(n\): the number of observations or measurements, \(\bar{x}\): average value of parameter \(x\), \(x_i\): the value of a discrete measurement.

- The standard deviation \(s\), which is determined using

\[
s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}
\]

A larger value of the standard deviation corresponds to a high degree of scatter.
Example Problem 6.6

The following tensile strengths were measured for four specimens of the same steel alloy:

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>520</td>
</tr>
<tr>
<td>2</td>
<td>512</td>
</tr>
<tr>
<td>3</td>
<td>515</td>
</tr>
<tr>
<td>4</td>
<td>522</td>
</tr>
</tbody>
</table>

(a) Compute the average tensile strength. (b) Determine the standard deviation.

Figure 6.20 (a) Tensile strength data, (b) The manner in which these data could be plotted. The data point corresponds to the average value of the tensile strength (TS); error bars that indicate the degree of scatter correspond to the average value plus and minus the standard deviation (TS ± s).
Example Problem 6.6

(a) \[ TS = \frac{\sum_{i=1}^{4} (TS)_i}{n} \]

\[ \Rightarrow TS = \frac{520 + 512 + 515 + 522}{4} = 517(MPa) \]

(b) \[ s = \sqrt{\frac{\sum_{i=1}^{4} [(TS)_i - TS]^2}{n-1}} \]

\[ \Rightarrow s = \sqrt{\frac{(520 - 517)^2 + (512 - 517)^2 + (515 - 517)^2 + (522 - 517)^2}{4 - 1}} \]

\[ \Rightarrow s = 4.6(MPa) \]
**Design/safety factors**

- Three will always be uncertainties in characterizing the magnitude of applied loads and their associated stress levels for in-service applications. Virtually all engineering materials exhibit a variability in their measured mechanical properties.

- For less critical static situations and when tough materials are used, a design stress, $\sigma_d$, is taken as the calculated stress level $\sigma_c$ (on the basis of the estimated maximum load) multiplied by a design factor, $N'$.

\[
\sigma_d = N' \sigma_c
\]

$N'$: design factor ($> 1$), $\sigma_c$: estimated maximum load.

The material to be used for the particular application is chosen so as to have a yield strength at least as high as this value of $\sigma_d$.

- The safe stress or working stress, $\sigma_w$, is based on the yield strength of the material and is defined as the yield strength ($\sigma_y$) divided by a factor of safety, $N$.

\[
\sigma_w = \frac{\sigma_y}{N}
\]
Utilization of design stress is usually preferred because it is based on the anticipated maximum applied stress instead of the yield strength of the material.

The choice of an appropriate value of $N$ is necessary. If $N$ is too large, then component overdesign will result; that is, either too much material or an alloy having a higher-than-necessary strength will be used. Values normally range between 1.2 and 4.0.

Selection of $N$ will depend on a number of factors, including economics, previous experience, the accuracy with which mechanical forces and material properties may be determined, and the consequences of failure in terms of loss of life and/or property damage.
A tensile-testing apparatus is to be constructed that must withstand a maximum load of 220,000 N. The design calls for two cylindrical support posts, each of which is to support half of the maximum load. Furthermore, plain-carbon (1045) steel ground and polished shafting rounds are to be used; the minimum yield and tensile strengths of this alloy are 310 MPa and 565 MPa, respectively. Specify a suitable diameter for these support posts. (Suppose a factor of safety: 5)

\[
\sigma_w = \frac{\sigma_y}{N} = \frac{310 \text{ MPa}}{5} = 62 \text{ MPa}
\]

\[
\Theta \ A_0 = \pi \left( \frac{d}{2} \right)^2
\]

\[
\Theta \ \sigma_w = \frac{F}{A_0} \Rightarrow A_0 = \frac{F}{\sigma_w}
\]

\[
\therefore \ \pi \left( \frac{d}{2} \right)^2 = \frac{F}{\sigma_w} \Rightarrow d = 2 \times \sqrt{\frac{F}{\pi \sigma_w}}
\]

\[
d = 2 \times \sqrt{\frac{110,000 \text{ N}}{\pi \times (62 \times 10^6 \text{ N/m}^2)}} = 4.75 \times 10^{-2} \text{ m} = 47.5 \text{ mm}
\]
Materials Selection for a Torsionally Stressed Shaft

\[ \tau = \frac{M_t \cdot r}{J} \]

\[ J = \frac{\pi \cdot r^4}{2} \Rightarrow \tau = \frac{2M_t}{\pi \cdot r^3} \]

\[ \tau_f = \frac{2M_t}{N} \]

\[ m = \pi r^2 \cdot L \cdot \rho \Rightarrow r = \sqrt{\frac{m}{\pi \cdot L \cdot \rho}} \]

\[ \tau_f = \frac{2M_t}{N} = 2M_t \cdot \sqrt{\frac{\pi \cdot L^3 \cdot \rho^3}{m^3}} \]

\[ \Rightarrow m = (2NM_t)^{2/3} \cdot (\pi^{1/3} L) \cdot \left( \frac{\rho}{\tau_f^{2/3}} \right) \]

Performance index \( P \)

\[ P = \frac{\tau_f^{2/3}}{\rho} \]

\[ \Rightarrow \log \tau_f = \frac{3}{2} \log \rho + \frac{3}{2} \log P^{76} \]
A plot of log $\tau_f$ versus log $\rho$ will yield a family of straight and parallel lines all having a slope of 3/2; each line in the family corresponds to a different performance index $P$. These lines are termed design guidelines, and four have been included in Fig. 6.22 for $P$ values of 3, 10, 30, and 100 (MPa)$^{2/3}$m$^3$/Mg.

**Design Guideline**

Figure 6.22 Strength versus density materials selection chart. Design guidelines for performance indices of 3, 10, 30, and 100 (MPa)$^{2/3}$m$^3$/Mg have been constructed, all having a slope of 3/2.
Materials Selection for a Torsionally Stressed Shaft

Selection Rules

• Performance index
  - For the sake of argument let us pick \( P = 10 \text{(MPa)}^{2/3} \text{m}^3/\text{Mg}. \)
  - Candidates: wood products, some plastics, some engineering alloys, engineering composites, glasses, and engineering ceramics.

• Toughness
  - Engineering ceramics and glasses are ruled out.
  - Candidates: wood products, some plastics, some engineering alloys, and engineering composites.

• Strength
  - It must equal or exceed 300 MPa.
  - Candidates: steels, titanium alloys, high-strength aluminum alloys, and engineering composites.
Materials Selection for a Torsionally Stressed Shaft

Selection Rules

• Cost
  - In real-life engineering situations, economics of the application often is the overriding issue and normally will dictate the material choice.

• Other issues
  Stiffness, fatigue strength, fabrication costs.


Materials Selection for a Torsionally Stressed Shaft

- **Performance index**
  Woods, engineering polymers, engineering alloys, engineering composites, glasses, engineering ceramics.

- **Toughness**
  glasses and ceramics $\Rightarrow$ out.

- **Strength** $> 300$ MPa

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**Figure 6.23** Strength versus density materials selection chart. Those materials lying within the shaded region are acceptable candidates for a solid cylindrical shaft that has a mass-strength performance index in excess of $10 \text{ (MPa)}^{2/3} \text{m}^3/\text{Mg}$, and a strength of at least $300$ MPa.
### Materials Selection for a Torsionally Stressed Shaft

#### Selection Rules

- **Price per mass per performance index**
  
  4340 steel < glass fiber-reinforced composite < 2024-T6 aluminum alloy < carbon fiber-reinforced composite < Ti-6Al-4V titanium alloy.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (Mg/m$^3$)</th>
<th>$\tau_f$ (MPa)</th>
<th>$P$ (MPa)$^{\frac{2}{3}}$m$^3$/Mg</th>
<th>$C$ (price/mass)</th>
<th>$\frac{C}{P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon fiber-reinforced composite (0.65 fiber fraction)</td>
<td>1.5</td>
<td>1140</td>
<td>72.8</td>
<td>80</td>
<td>1.10</td>
</tr>
<tr>
<td>Glass fiber-reinforced composite (0.65 fiber fraction)</td>
<td>2.0</td>
<td>1060</td>
<td>52.0</td>
<td>40</td>
<td>0.77</td>
</tr>
<tr>
<td>Aluminum alloy (2024-T6)</td>
<td>2.8</td>
<td>300</td>
<td>16.0</td>
<td>15</td>
<td>0.94</td>
</tr>
<tr>
<td>Titanium alloy (Ti-6Al-4V)</td>
<td>4.4</td>
<td>525</td>
<td>14.8</td>
<td>110</td>
<td>7.43</td>
</tr>
<tr>
<td>4340 steel (oil-quenched and tempered)</td>
<td>7.8</td>
<td>780</td>
<td>10.9</td>
<td>5</td>
<td>0.46</td>
</tr>
</tbody>
</table>